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# Beyond Accuracy and Alignment: A Diagnostic Evaluation Protocol for Feedback Alignment

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## Abstract

1 Modern feedback-alignment evaluation on deep residual networks is still summa-  
2 rized by a deceptively simple pair: headline accuracy and headline cosine align-  
3 ment  $\Gamma$  to the backpropagation gradient. We show that this pair can silently fail in  
4 two distinct ways on standard CIFAR-10 pre-LayerNorm ResMLP and ViT-Mini  
5 settings: first, *measurement degeneracy*, where residual-stream growth drives  
6 hidden-layer BP gradients to the numerical floor and makes  $\Gamma$  uninterpretable;  
7 and second, *low intrinsic credit-direction quality*, where random-feedback credit  
8 remains essentially unaligned with BP on the deep blocks even when the reference  
9 gradient is still meaningful. The headline result is that the field-standard reporting  
10 pair walks back none of the methods we audit, whereas a four-diagnostic proto-  
11 col walks back the three degenerate methods and passes the two trustworthy con-  
12 trols. Intervention with a per-block scale-control penalty further reveals method-  
13 dependent severity within the audited fixed-feedback family: State Bridge then  
14 exceeds the architecture-matched frozen-blocks baseline by about 10 percentage  
15 points, while Credit Bridge attains much higher deep BP cosine than DFA at the  
16 same final accuracy, a dissociation that motivates reporting layerwise credit quality  
17 jointly with a depth-utilization baseline. Our contribution is an evaluation method-  
18 ology paper for the NeurIPS 2026 Evaluations & Datasets track: we provide the  
19 protocol, the calibration logic for its thresholds, a reference implementation, a five-  
20 method audit, and validation through temporal replay, cross-architecture checks,  
21 intervention-based disambiguation, and a documented catalog of pipeline pitfalls,  
22 in the spirit of critical evaluation analyses such as Jordan et al. [3], O’Bray et al.  
23 [2], Paleka et al. [1].

## 24 1 Introduction

25 Backpropagation (BP) is the de facto training method for deep neural networks, but its requirement  
26 that each feedback connection carry a weight identical to the corresponding forward connection –  
27 the weight-transport problem – has long been considered biologically implausible [4, 8]. *Feedback*  
28 *alignment* (FA) [4] side-steps weight transport by delivering per-layer credit through fixed random  
29 feedback matrices, and its direct variant (DFA) [5] projects the output error to every hidden layer  
30 through an independent random matrix; parallel lines include target propagation [15] and equilib-  
31 rium propagation [9]. These rules are studied both as biologically-plausible alternatives to BP and  
32 as scalable, asynchronous training schemes, with recent work scaling DFA to transformer-scale ar-  
33 chitectures on language, recommendation, and view-synthesis tasks [7, 6]. Evaluation in this line of  
34 work has converged on a two-number summary: final task accuracy, and an aggregate cosine align-  
35 ment  $\Gamma$  between the method’s per-layer credit and the BP gradient on the trained network [4–8].

36 On the audited 4-block  $d=256$  ResMLP, however, Table 1 already shows that this accuracy-plus- $\Gamma$   
 37 pair is not a validity check: DFA reaches only  $0.306 \pm 0.006$  test accuracy, below the architecture-  
 38 matched frozen-blocks baseline of  $0.349 \pm 0.002$ , while still looking superficially comparable to  
 39 other non-BP methods. Figure 1 further shows that the apparent cosine evidence is concentrated  
 40 at the shallowest block, with DFA at seed 42 reaching about  $+0.42$  at layer 0 but approximately  
 41  $-0.03$  to  $0$  on layers 1–4, so the aggregate obscures where credit direction is and is not present. At  
 42 the same time, the deepest BP reference norm is only about  $5 \times 10^{-10}$  for DFA, State Bridge, and  
 43 Credit Bridge, below the  $10^{-8}$  clamp used by `F.cosine_similarity`, whereas BP remains around  
 44  $4 \times 10^{-4}$ , so the reported deep cosine is partly computed against a numerical-floor reference rather  
 45 than an informative gradient direction (Figure 1; Table 1). Those numbers can be useful, but only if  
 46 the measurement regime itself is valid.

47 Our audit shows that modern residual vision models can make these two quantities look informative  
 48 while failing to answer the question they are taken to answer. Figure 1 shows the first failure mode,  
 49 which we call *Mode 1: measurement degeneracy*, where residual-stream growth drives the deepest  
 50 hidden state to about  $\|h_L\| \sim 10^8$  under DFA/SB/CB while the corresponding BP reference col-  
 51 lapses to  $\|g_L\| \sim 5 \times 10^{-10}$ , so the deep-layer cosine is measured against a clamp-dominated floor  
 52 rather than a meaningful target direction. The same figure also shows the second failure mode, *Mode*  
 53 *2: low intrinsic credit-direction quality*, because even after comparing against the stronger frozen-  
 54 blocks baseline ( $0.349 \pm 0.002$ ) and looking layer-by-layer, DFA’s deep blocks remain essentially  
 55 null while only layer 0 is visibly positive. Intervention sharpens both modes. Adding a per-block  
 56 residual penalty  $\lambda \|f_i(h_i)\|^2$  to DFA at  $\lambda=10^{-2}$  contains  $\|h_L\|$  to about  $4 \times 10^4$  and lifts the deep BP  
 57 reference to about  $10^{-6}$ , but DFA’s rescued deep cosine is only about  $+0.16$ ; State Bridge under the  
 58 same intervention reaches a three-seed deep cosine of  $+0.32$  and, unlike DFA, exceeds the frozen-  
 59 blocks baseline by  $+10$  points in final accuracy; Credit Bridge reaches a deep cosine near  $+0.68$   
 60 yet matches only the DFA accuracy, so Mode 2 has method-dependent severity and deep cosine is  
 61 not a sufficient predictor of final accuracy across methods. At the same time, at  $\lambda=10^{-4}$  Mode 1 is  
 62 alleviated while the DFA deep cosine still stays near zero, and at vanilla DFA epoch 1 the reference  
 63 is already meaningful at about  $6 \times 10^{-7}$  but the deep cosine is still  $-0.008 \pm 0.013$  across three  
 64 seeds. The failure is therefore neither unitary nor uniform: Mode 1 and Mode 2 are observationally  
 65 separable, and within the audited fixed-feedback family, the severity of each mode varies by method.

66 Accordingly, this paper does not introduce a new FA variant or a new benchmark. Of the five  
 67 methods we audit, BP, EP, and DFA are established baselines from the published literature; the  
 68 remaining two, which we call *State Bridge* and *Credit Bridge*, are diagnostic probes we construct  
 69 in this paper to directly learn the two targets that different strands of the BP-free literature argue  
 70 should produce good per-layer credit (formal definitions and citations in Section 2). Instead, Table 1  
 71 and Figure 1 use a standard five-method CIFAR-10 audit to show that status-quo reporting would  
 72 treat BP, EP, DFA, State Bridge, and Credit Bridge as the same kind of evidence-bearing object  
 73 even though only BP and EP remain trustworthy under matched diagnostic checks. This makes the  
 74 contribution methodological in the sense of Jordan et al. [3], O’Bray et al. [2], and Paleka et al. [1]:  
 75 the central question is not whether one more FA variant can post a headline number, but whether the  
 76 reporting pipeline distinguishes meaningful credit-direction evidence from numerical-floor artifacts  
 77 and from shallow-only learning. The protocol therefore starts from per-layer diagnostics and a  
 78 frozen-blocks baseline before reading any aggregate cosine or final accuracy as evidence about deep  
 79 credit assignment. We first show the walk-back on a standard audit, then isolate the two failure  
 80 modes, and finally state the reporting protocol that future FA papers should satisfy.

## 81 2 Audit: Standard Reporting Walks Back Nothing

82 Table 1 fixes the canonical audit to a 4-block pre-LayerNorm ResMLP with width  $d=256$  on CIFAR-  
 83 10, trained for 100 epochs with AdamW (learning rate  $10^{-3}$ , weight decay 0.01), a cosine schedule,  
 84 batch size 128, and three seeds (42, 123, 456); all five methods are read against the identical ar-  
 85 chitecture, optimizer, schedule, and training budget without method-specific tuning, and Figure 1  
 86 summarizes the corresponding per-block growth, deepest-layer BP reference norm, cross-batch sta-  
 87 bility, and frozen-baseline comparison.

88 Two rows in Table 1, *State Bridge* (SB) and *Credit Bridge* (CB), are diagnostic probes we  
 89 construct in this paper, not prior FA variants. Each directly learns a target that a different  
 90 strand of the BP-free literature argues should produce good per-layer credit, and each uses the

Table 1: Main audit table for the 4-block  $d=256$  pre-LayerNorm ResMLP on CIFAR-10. The row and column structure is fixed here; fill from the three-seed audit output.

Method	Test acc.	Headline $\Gamma$	Status-quo verdict	Protocol verdict
BP	$0.615 \pm 0.003$	$\approx 1.0$	trustworthy	trustworthy
EP	$0.316 \pm 0.030$	0.008	trustworthy	trustworthy
DFA	$0.306 \pm 0.006$	0.10	trustworthy	walked back
State Bridge	$0.205 \pm 0.032$	0.005	trustworthy	walked back
Credit Bridge	$0.289 \pm 0.026$	0.07	trustworthy	walked back

91 same block local loss  $-\langle f_l(h_l), a_l \rangle$  as DFA but with a different  $a_l$ . SB instantiates the target-  
 92 propagation view that accurate prediction of a downstream hidden state yields a usable credit  
 93 signal [14, 15]: an auxiliary  $G_\psi(h_l, t_l, s)$  is fit by MSE to predict  $h_L$  from  $(h_l, t_l=l/L, s=e_T)$ ,  
 94 and  $a_l^{\text{SB}} = \nabla_{h_l} \text{CE}(W_{\text{out}} \text{LN}(G_\psi(h_l, t_l, s)), y)$ . CB instantiates the synthetic-gradient view that a  
 95 learned value network, if its input-gradient approximates the BP gradient, can stand in for it [16]:  
 96  $V_\phi(h_l, t_l, s)$  is fit via a bridge residual against an EMA target, and  $a_l^{\text{CB}} = \nabla_{h_l} V_\phi(h_l, t_l, s)$ . Both  
 97 auxiliaries are trained on detached hidden states. We use SB and CB as controls that populate differ-  
 98 ent points in the (angular agreement with BP, functional usefulness) plane; that is what makes the  
 99 cross-method cosine-versus-accuracy dissociation in Section 4 visible.

100 By the field’s usual criteria, the non-BP methods appear to train to nontrivial accuracy and report  
 101 nonzero alignment. In Table 1, DFA reaches  $0.306 \pm 0.006$  test accuracy with headline  $\Gamma=0.10$ ,  
 102 State Bridge reaches  $0.205 \pm 0.032$  with  $\Gamma=0.005$ , and Credit Bridge reaches  $0.289 \pm 0.026$  with  
 103  $\Gamma=0.07$ ; none of these rows looks like an obvious invalidation if one is reading the usual pair of final  
 104 accuracy and aggregate alignment in the style of prior FA reporting [4–7]. Even the absolute scale  
 105 does not itself force a walk-back, because all three methods are plainly above chance and all three  
 106 report positive headline alignment rather than a visibly broken or undefined quantity. That reading  
 107 is exactly what the rest of the paper overturns.

108 Low accuracy by itself is not the pathology. Equilibrium Propagation (EP), a contrastive energy-  
 109 based alternative to BP that updates weights from the difference between a free-phase and a nudged-  
 110 phase hidden trajectory, is the key internal comparison in Table 1 and Figure 1: it achieves only  
 111  $0.316 \pm 0.030$  accuracy and a very small headline  $\Gamma=0.008$ , yet its per-block growth is only  $11.6\times$ ,  
 112 its deepest BP reference norm remains around  $1.3 \times 10^{-4}$  rather than collapsing to the numerical  
 113 floor, and its cross-batch direction-stability score is 0.02 rather than the much higher drift-dominated  
 114 values seen for DFA-family methods. At the same time, EP is not a positive result for depth usage  
 115 in the stronger sense, because its trainable-model accuracy is still 3.3 percentage points below the  
 116 frozen-blocks baseline of  $0.349 \pm 0.002$ . The distinction matters because it separates underperform-  
 117 ance from invalid evaluation.

118 When we compare each method to a frozen-blocks baseline matched to the same architecture, the  
 119 headline interpretation changes immediately. The frozen-blocks model, which trains only the em-  
 120 bedding, LayerNorm, and head while holding the residual blocks fixed, reaches  $0.349 \pm 0.002$  across  
 121 the same three seeds; against that baseline, BP is higher by 26.6 points, but DFA is lower by 4.3  
 122 points, State Bridge by 14.4 points, Credit Bridge by 6.0 points, and even EP by 3.3 points. Fig-  
 123 ure 1 shows that this accuracy comparison lines up with the diagnostic split: DFA, State Bridge, and  
 124 Credit Bridge also combine extreme per-block growth ( $237\times$ ,  $12000\times$ , and  $96\times$ ), deepest-layer BP  
 125 norms around  $10^{-9}$ , and high cross-batch instability (0.16, 0.53, and 0.37), so their deep blocks are  
 126 at best passengers and in practice often harmful. This establishes the audit question the rest of the  
 127 paper must answer: why do the standard signals fail so badly?

### 128 3 Failure Mode 1: Measurement Degeneracy

129 Mode 1 has two parts. The activation-growth part (a) is a scale pathology of fixed-feedback local-  
 130 credit objectives without an effective scale-control term: for block  $l$ , DFA, State Bridge, and Credit  
 131 Bridge each update  $f_l$  by reducing a local loss of the form  $-\langle f_l(h_l), a_l \rangle$ , where the per-layer credit  
 132 vector  $a_l$  is the method-specific projection of the output error (for DFA,  $a_l = B_l^\top e_T$  with a fixed

5-method audit on 4-block  $d=256$  ResMLP CIFAR-10 (3-seed mean  $\pm$  std)

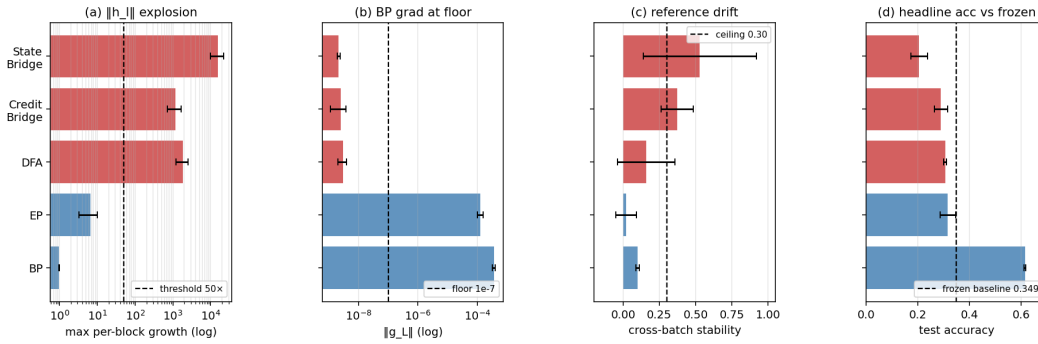


Figure 1: Five-method audit on the 4-block  $d=256$  pre-LayerNorm ResMLP: the field-standard pair looks superficially consistent across methods, but the diagnostic view separates trustworthy controls from walked-back methods.

133 random  $B_l$ ; for State Bridge,  $a_l$  is the gradient of a cross-entropy loss measured through a learned  
 134 state predictor  $G_\psi(h_l, t_l, s)$  that estimates  $h_L$ ; for Credit Bridge,  $a_l$  is the gradient of a learned  
 135 value network  $V(h_l, t_l, s)$ . None of these three local losses contains a penalty on  $\|f_l(h_l)\|$ , so  
 136 any direction in which a larger block output improves inner-product alignment with the method’s  
 137 fixed or learned credit target is rewarded; in a pre-LN residual stack, larger block outputs directly  
 138 increase residual-stream scale, and terminal LayerNorm at the output removes task-loss sensitivity  
 139 to that scale, so the architecture supplies no global restraint on the local growth incentive. The  
 140 gradient-floor part (b) follows from the LayerNorm Jacobian. For  $y = \text{LN}(h) = (h - \mu(h))/\sigma(h)$   
 141 with  $\sigma(h) = (\frac{1}{d} \sum_i (h_i - \mu(h))^2)^{1/2}$  proportional to  $\|h\|/\sqrt{d}$ , the spectral norm of  $\partial y/\partial h$  is  
 142  $\Theta(1/\sigma(h))$ , so back-propagating through terminal LayerNorm scales the deepest hidden BP  
 143 gradient as  $\|g_L\| = \Theta(1/\|h_L\|)$ , and the same residual-stream inflation that drives diagnostic (a) drives  
 144 a proportional collapse of the diagnostic (b) reference. Empirically, on the audited 4-block pre-  
 145 LayerNorm ResMLP ( $d=256$ , CIFAR-10, 100 epochs, 3 seeds), DFA training drives the three-seed  
 146 mean  $\|h_L\|$  from about 9 at initialization to about  $5 \times 10^8$  by epoch 100 and  $\|g_L\|$  from about  
 147  $9.8 \times 10^{-4}$  to about  $4 \times 10^{-10}$ , while the reported deep cosine remains defined only because  
 148  $F.\text{cosine\_similarity}$  clamps the denominator at  $\varepsilon=10^{-8}$  (Table 1; Figure 1). At that endpoint  
 149 the reference norm is about  $20\times$  below the clamp, so the quantity being reported is effectively  
 150  $(a \cdot b)/(\|a\| \max(\|b\|, 10^{-8}))$  rather than a comparison to a meaningful BP direction.

151 We tested this mechanism story against four natural alternative attributions, all of which it survives.  
 152 *Not residual-skip-driven*: with terminal LN kept and the additive skip removed ( $h_{l+1} = F_l(h_l)$ ), DFA  
 153 still converges to  $\|h_L\| \approx 1.06 \times 10^8$  and  $\|g_L\| \approx 1.09 \times 10^{-10}$  at 100 epochs, both at the diagnostic  
 154 floor (Appendix H). *Not task-signal-driven*: under i.i.d. random class targets per minibatch, DFA  
 155 still reaches  $\|h_L\| \approx 1.67 \times 10^8$  and  $\|g_L\| \approx 8 \times 10^{-12}$  while accuracy stays at chance (Appendix I). *Not*  
 156 *DFA-specific*: the same random-target ablation drives  $\|h_L\|$  to  $6.2 \times 10^3$  for SB and  $2.0 \times 10^4$  for CB  
 157 in three epochs, so all three audited fixed-feedback methods exhibit data-agnostic activation growth.  
 158 *Not shared by EP*: under the same protocol, EP keeps  $\|h_L\| \approx 586$  at five epochs,  $25\times$  smaller than  
 159 DFA’s three-epoch value, confirming that the random-target assay separates the explosion-prone  
 160 fixed-feedback class from EP’s energy-based objective.

161 The matched same-backbone causal control for diagnostic (b) is removing terminal LayerNorm. On  
 162 the same ResMLP-d256 with the residual skip intact, 100 epochs of DFA, three seeds, the residual  
 163 stream still inflates to  $\|h_L\| \approx 1.21 \times 10^7$ , but the deepest hidden-layer BP gradient remains at  
 164  $\|g_L\| \approx 7.2 \times 10^{-4}$  (four orders of magnitude above the diagnostic (b) floor), and the final test  
 165 accuracy is  $0.327 \pm 0.012$ , statistically indistinguishable from vanilla DFA’s  $0.306 \pm 0.006$  on the  
 166 same backbone with terminal LayerNorm intact. Removing terminal LayerNorm therefore preserves  
 167 Mode 1 (a) but cleanly eliminates Mode 1 (b) on the same architecture, while leaving final task  
 168 accuracy essentially unchanged. Combined with the broader cross-architecture pattern (StudentNet  
 169 and the BatchNorm CNN, which lack terminal LayerNorm, never trigger diagnostic (b); ViT-Mini  
 170 with a terminal LN does, by epochs 2–3 (Figure 2)), terminal LayerNorm is necessary for Mode 1 (b)

171 in the audited residual ResMLP and ViT-Mini setting. The collapse is also not a late-epoch curiosity:  
 172  $\|g_L\|$  drops from  $9.8 \times 10^{-4}$  at epoch 0 to  $6.7 \times 10^{-8}$  by epoch 4 in the temporal replay across three  
 173 seeds, so the protocol fires within the first 11 epochs of a 100-epoch run and is actionable as an  
 174 early-stop criterion rather than a post hoc explanation. Once measurement degeneracy is identified,  
 175 the next question is whether poor deep credit remains even before collapse.

#### 176 4 Failure Mode 2: Low Intrinsic Credit-Direction Quality

177 The second failure mode appears even in the meaningful-measurement regime. At the earliest vanilla  
 178 DFA checkpoints on ResMLP, the hidden backpropagated gradient at the first deep block remains  
 179 above the numerical floor: at epoch 1,  $\|g_2\|$  is  $6.8 \times 10^{-7}$ ,  $6.6 \times 10^{-7}$ , and  $3.8 \times 10^{-7}$  across the three  
 180 seeds, all above the  $10^{-7}$  threshold used to distinguish measurable from collapsed gradients. Yet the  
 181 corresponding deep-layer cosine values are already essentially null: across layers 1–4, all seed-level  
 182 measurements at epoch 1 lie in  $[-0.04, +0.02]$ , with a three-seed mean of  $-0.008 \pm 0.013$ , and  
 183 by epoch 2 the deep mean is still only  $-0.018 \pm 0.018$  (Table 2). This is the observational pattern  
 184 predicted by low credit-direction quality rather than mere disappearance of signal: the gradient is  
 185 still present enough to measure, but the directions delivered to the deep network carry little agree-  
 186 ment with backpropagation, consistent with prior concerns that alternative feedback rules can fail  
 187 by supplying poor credit assignments even before full collapse [8, 10, 12, 11]. This rules out the  
 188 simplest objection that the deep-layer null result is merely a byproduct of collapse.

189 A second metric with different numerical failure modes tells the same story. Cosine measures direc-  
 190 tional agreement with the BP gradient, whereas the per-layer perturbation correlation  $\rho_l$  measures  
 191 whether the proposed credit predicts the actual loss response: for  $M=32$  unit-norm random di-  
 192 rections  $v_m$  and step  $\varepsilon=10^{-3}$ ,  $\rho_l = \text{Pearson}_m(\langle a_l, \varepsilon v_m \rangle, \ell(h_l + \varepsilon v_m) - \ell(h_l))$ , evaluated per  
 193 sample on a fixed eval batch and then averaged. Cosine and  $\rho$  have different failure modes, espe-  
 194 cially with respect to normalization and small-denominator effects. In our controls,  $\rho$  behaves as  
 195 expected, with a Taylor-ceiling positive control near  $+0.997$  and a random-vector negative control  
 196 near  $+0.006$  (Figure 3, Table 2). On vanilla DFA, deep  $\rho$  is likewise null: for the early checkpoints  
 197 where the gradients remain measurable, the deep average is  $-0.003 \pm 0.005$  across seeds and epochs,  
 198 and in a floor-level checkpoint it is  $+0.002$ , again indistinguishable from noise. The agreement be-  
 199 tween cosine and  $\rho$  therefore rules out the interpretation that the null deep result is an artifact of  
 200 cosine’s  $\varepsilon$ -clamp or vector normalization. The deep blocks are not just hard to measure; they are  
 201 receiving weakly useful directions.

202 Per-layer reporting is therefore not cosmetic. In ResMLP under vanilla DFA, the headline aggregate  
 203 alignment  $\Gamma \approx 0.07$ – $0.10$  can look mildly positive only because layer 0 remains strongly aligned  
 204 while the deep network is not: at the same epoch-1 checkpoints where layers 1–4 are essentially zero,  
 205 layer 0 has cosine  $+0.42$ ,  $+0.44$ , and  $+0.42$  across seeds (Table 2; per-seed values in Appendix K).  
 206 The resulting average can therefore be driven by the embedding layer even when the interior blocks  
 207 are effectively unaligned, so aggregate reporting obscures the very distinction needed to separate  
 208 “measurement collapse” from “poor credit direction.” This layer-0 dominance is specific to the  
 209 ResMLP DFA setting; on ViT-Mini DFA, all layers are near zero, which strengthens the broader  
 210 methodological point that alignment should be reported per layer rather than only in aggregate. With  
 211 the two modes separated observationally, the remaining question is whether intervention can move  
 212 them independently.

213 Mode 2 has method-dependent severity within the audited fixed-feedback family once Mode 1 is  
 214 alleviated. Applying the same per-block scale-control penalty  $\lambda=10^{-2}$  that rescued DFA to State  
 215 Bridge and to Credit Bridge on the same 4-block  $d=256$  ResMLP backbone over 30 epochs and three  
 216 seeds gives converged test accuracies of  $0.453 \pm 0.003$  (SB) and  $0.360 \pm 0.003$  (CB), with deep mean  
 217 cosines of  $+0.322 \pm 0.007$  (SB) and  $+0.679 \pm 0.008$  (CB) and deep mean  $\rho$  of  $+0.402 \pm 0.015$   
 218 (SB) and  $+0.464 \pm 0.025$  (CB), while DFA under the same intervention reaches  $0.360 \pm 0.001$   
 219 with deep cosine  $+0.151 \pm 0.025$  and deep  $\rho$   $+0.080 \pm 0.011$  (Table 2; Appendix J). The State  
 220 Bridge penalty rescue is roughly 24 percentage points above the vanilla State Bridge baseline of  
 221  $0.213$  on the same architecture and, more importantly for the paper’s central walk-back, exceeds  
 222 the architecture-matched frozen-blocks shallow baseline of  $0.349$  by  $+10.4$  percentage points. State  
 223 Bridge with the penalty intervention is therefore the first audited non-BP method whose trained deep  
 224 blocks substantively improve over an architecture-matched random-block baseline; the headline ac-  
 225 curacy gap is comparable to BP+penalty’s  $+18.1$  pp over the same shallow baseline. Neither the

Table 2: Two-mode validation table built around the intervention and disambiguation results.

Condition	Deep-layer alignment signal	Measurement regime	Interpretation
Vanilla DFA, early epoch	$\overline{\text{cos}}_{deep} = -0.008 \pm 0.013, \overline{\rho}_{deep} = -0.003 \pm 0.005$	meaningful ( $\ g\  \sim 10^{-6}$ )	mode 2 present without mode 1
Vanilla DFA, converged	$\overline{\text{cos}}_{deep} = -0.022, \overline{\rho}_{deep} = +0.002$	degenerate ( $\ g\  \sim 10^{-9}$ )	mode 1 obscures mode 2
Penalized DFA, $\lambda = 10^{-2}$	$\overline{\text{cos}}_{deep} = +0.151 \pm 0.025, \overline{\rho}_{deep} = +0.080 \pm 0.011$	meaningful ( $\ g\  \sim 10^{-6}$ )	partial alleviation of both modes
Fresh- $B$ null control	$\overline{\text{cos}}_{deep} = +0.002 \pm 0.022$ ( $n=20$ draws)	meaningful	training-specific adaptation check

226 activation scale nor the deep BP gradient magnitude is silenced under the penalty:  $\|h_L\|$  stays at  
 227  $302 \pm 8$  for SB and  $5680 \pm 178$  for CB, with  $\|g_L\|$  at  $\sim 1.8 \times 10^{-4}$  and  $\sim 1.9 \times 10^{-5}$  respectively,  
 228 both well within the meaningful-measurement regime, so the recovered deep cosines are computed  
 229 against an informative reference and not against a numerical floor. Within this rescued regime, the  
 230 three methods reveal a clean cosine-versus-accuracy dissociation. Credit Bridge achieves roughly  
 231  $4\times$  the deep cosine of DFA and  $2\times$  that of State Bridge, yet its final accuracy matches DFA’s and  
 232 is 9 percentage points below State Bridge’s. We therefore frame the Mode 2 reading as a three-part  
 233 proposition. *Observation*: under the same intervention and matched training budget, CB and DFA  
 234 reach the same accuracy despite a  $4\times$  deep-cosine gap, while SB is the best accuracy with interme-  
 235 diate cosine. *Inference*: layerwise cosine to the BP gradient is necessary to rule out grossly wrong  
 236 credit signals (it distinguishes the rescued regime from the clamp-dominated vanilla regime), but  
 237 it is not sufficient to certify that the supplied signal is useful credit for depth. *Mechanism hypoth-*  
 238 *esis*: usefulness depends on whether the local update induces useful forward-state change across  
 239 blocks, not merely whether its direction is close to the BP gradient in angle. Under this reading, CB  
 240 supplies a gradient-direction surrogate that aligns with BP in angle but does not translate to a coordi-  
 241 nated forward-state improvement, while State Bridge supplies a state-level downstream teaching  
 242 signal that preserves aspects of useful credit which layerwise cosine does not measure. We state this  
 243 as a mechanism hypothesis rather than a theorem because we have measured the angle-to-accuracy  
 244 gap but not the full functional-credit content; the reporting rule that follows is robust to either inter-  
 245 pretation. This cross-method dissociation strengthens the methodological point that alignment must  
 246 be reported jointly with measurement validity and a depth-utilization baseline rather than as a single  
 247 headline number.

## 248 5 Intervention and Cross-Architecture Evidence

249 The penalty intervention first matters as a rescue of the measurement regime. When we add a per-  
 250 block penalty  $\lambda \text{mean}(\|f_i(h_i)\|^2)$  to DFA’s local loss and train the 4-block  $d=256$  ResMLP for 30  
 251 epochs on CIFAR-10, the  $\lambda=10^{-2}$  setting contains the terminal hidden-state scale from  $\|h_L\| \sim$   
 252  $4.4 \times 10^8$  under vanilla DFA to  $\sim 4.0 \times 10^4$ , while lifting the deepest BP reference norm from  
 253  $\|g_L\| \sim 5 \times 10^{-10}$  to  $\sim 9.0 \times 10^{-7}$ , a roughly four-order-of-magnitude rescue on both quantities  
 254 (Figure 3; Table 2). At that setting, both diagnostic (a) and diagnostic (b) pass on penalized DFA, and  
 255 test accuracy rises to  $0.360 \pm 0.001$  from  $0.301 \pm 0.005$  for matched 30-epoch vanilla DFA. The key  
 256 point is not yet that the recovered network has good deep credit, but that the deep reference vector  
 257 is again large enough to function as a meaningful target direction rather than a clamp-dominated  
 258 artifact. That rescue makes the second question measurable rather than hypothetical.

259 Once the reference vector is meaningful again, the deep layers no longer sit exactly at null. At  
 260  $\lambda=10^{-2}$ , penalized DFA reaches a three-seed deep-layer mean cosine of  $+0.151 \pm 0.025$  and deep  
 261 perturbation correlation of  $+0.080 \pm 0.011$ , whereas vanilla DFA is essentially zero on both metrics  
 262 in the deep blocks, consistent with prior concerns that alternative feedback can fail by supplying  
 263 poor credit directions even before full collapse [8, 10, 12, 11]. The null calibration rules out the  
 264 interpretation that this recovered signal is merely measurement noise: on the same penalized check-  
 265 point, replacing the training-time feedback matrices with 20 fresh random  $B_l$  draws gives a deep  
 266 cosine of only  $+0.002 \pm 0.022$ , with per-layer standard deviations of 0.013–0.023, all within noise  
 267 of zero (Table 2). The  $\lambda$  sweep sharpens the dissociation further: at  $\lambda=10^{-4}$ , Mode 1 is already  
 268 alleviated, with three-seed mean  $\|h_L\| \approx 2.2 \times 10^4$  and  $\|g_L\| \approx 7.0 \times 10^{-7}$ , but the three-seed deep  
 269 cosine remains  $-0.020$ , while  $\lambda=10^{-2}$  delivers the  $+0.151$  and  $+0.080$  above (Figure 3). The  
 270 improvement is real, but it is only partial.

271 A rescue intervention is only informative if its direct cost is controlled. The relevant control is  
 272 BP trained under the same penalty for the same matched 30-epoch budget: across three seeds, BP  
 273 falls from  $0.585 \pm 0.001$  without the penalty to  $0.530$  with  $\lambda=10^{-2}$  (BP+penalty single seed), so

Table 3: Protocol definition table. Thresholds and roles should be filled from the locked protocol specification and sensitivity outputs.

Diag.	Measurement	Default threshold	Role
(a)	Per-layer activation scale via max-per-block growth $\max_l \ h_{l+1}\ /\ h_l\ $	$> 50\times$	binary detector
(b)	Deepest hidden-layer BP gradient norm $\ g_L\ $	$< 10^{-7}$	binary detector
(c)	Cross-batch direction stability of normalized BP gradients	$> 0.30$	sub-mode discriminator
(d)	Frozen-blocks baseline margin for trained blocks over random blocks	$< 2\text{pp}$	depth-utilization check

274 the penalty has a direct cost of about 5.5 percentage points even when credit assignment is correct,  
 275 whereas DFA moves in the opposite direction, from  $0.301 \pm 0.005$  to  $0.360 \pm 0.001$ , and State Bridge  
 276 moves further still, from 0.213 to  $0.453 \pm 0.003$ , all under the same 30-epoch intervention (Figure 3;  
 277 Appendix J). Relative to the frozen-blocks baseline of 0.349, BP+penalty retains a margin of +18.1  
 278 points, State Bridge+penalty retains +10.4 points, and DFA+penalty retains only +1.1 points. The  
 279 remaining BP-to-DFA gap of 17.0 points is therefore a lower bound on the part of DFA’s deficit  
 280 that is not explained by simple penalty-induced capacity loss alone, though not a clean isolation  
 281 because BP uses an end-to-end loss whereas DFA uses block-local losses. The substantially smaller  
 282 BP-to-State-Bridge gap of  $0.530 - 0.453 = 7.7$  points shows that the cross-method differences in  
 283 penalty-rescued accuracy are not all attributable to a uniform “random-feedback ceiling”: the bridge  
 284 construction in State Bridge can recover much more of the BP-with-penalty performance than DFA  
 285 can, on the same architecture and the same intervention. The residual gap after that control is what  
 286 keeps Mode 2 substantively alive while letting it have method-dependent severity.

287 The architecture comparison sharpens the scope of the critique. In the terminal-LN architectures we  
 288 audited, both diagnostics fire for DFA-trained ResMLP at  $d=256$ , the same pattern recurs at  $d=512$   
 289 with even larger max-per-block growth (about  $1.5 \times 10^4$ ), and ViT-Mini with a class token and ter-  
 290 minal LN shows diagnostic (a) by epoch 1 and diagnostic (b) by epochs 2–3 (Figure 2). A depth  
 291 sweep on the  $d=512$  ResMLP at  $L \in \{2, 4, 6, 8, 12\}$  shows that the layerwise pattern is essentially  
 292 depth-invariant: DFA’s layer-0 cosine stays in  $[+0.39, +0.40]$  across all five depths, while its mean  
 293 deep-layer cosine stays within  $[-0.005, +0.000]$  and its deep perturbation correlation collapses to  
 294 0.000 in every depth tested, even though BP retains a deep-layer cosine of +0.94 at  $L=12$  (Ap-  
 295 pendix G). The deep credit signal does not improve when the network is shallower, so the failure  
 296 is not a “too deep” artifact. In the non-terminal-LN controls, the pattern is different: StudentNet  
 297 shows diagnostic (a) only at epochs 14–25 while diagnostic (b) never fires across 100 epochs and  
 298 three seeds, and the BatchNorm CNN on CIFAR-10 likewise shows strong growth under DFA, with  
 299 max-per-block growth up to  $237\times$ , but keeps deepest BP gradients around  $\|g\| \sim 10^{-3}$  and never  
 300 triggers diagnostic (b) (Figure 2). BP never triggers either diagnostic in any audited architecture.  
 301 The matched same-backbone ResMLP-d256 ablation in Section 3 supplies the cleanest causal con-  
 302 trol: removing terminal LayerNorm from the same architecture preserves activation growth but elim-  
 303 inates the gradient floor, so diagnostic (b) is necessary on terminal-LN ResMLP and is not just an  
 304 architecture-class coincidence. The broader claim therefore holds at full strength inside the audited  
 305 residual ResMLP and ViT-Mini regime, while diagnostic (a) remains useful more broadly. This lets  
 306 the paper end with a reporting rule rather than an overclaimed theory.

## 307 6 Recommended FA Evaluation Protocol

308 The reporting protocol begins with measurement validity. Before any FA paper reports a headline  
 309 alignment number, it should report per-layer state scale and the hidden BP reference-gradient scale  
 310 at the layers where the scientific claim is being made. In our audited regime, those two quantities  
 311 already separate healthy from invalid measurement with unusually wide margins: the maximum  
 312 per-block growth stays below about  $11\times$  for BP and EP but is at least  $694\times$  for the degenerate  
 313 methods, giving a  $63\times$  calibration gap, while the deepest hidden BP norm stays above about  $10^{-4}$   
 314 for BP and EP but below about  $4 \times 10^{-9}$  for the degenerate methods, giving a  $24,338\times$  gap (Table 3;  
 315 Table 1; Figure 4). These are not cosmetic diagnostics around the real result: they determine whether  
 316 the reported cosine is being computed against an informative BP direction or against a floor-level  
 317 reference. If the reference gradient is at floor, the evaluator should stop treating aggregate alignment  
 318 as evidence.

Cross-architecture temporal evolution of FA diagnostics (seed 42)

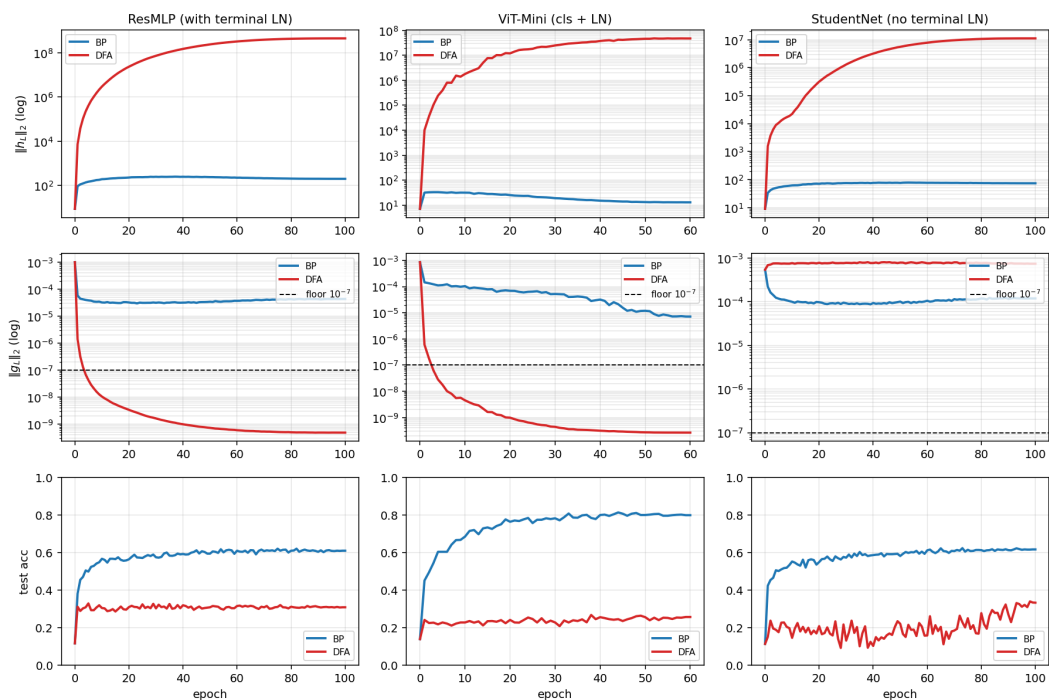


Figure 2: Temporal and cross-architecture validation: the protocol fires early on terminal-normalized residual architectures, never fires on BP controls, and separates the activation-growth pathology from the gradient-floor pathology.

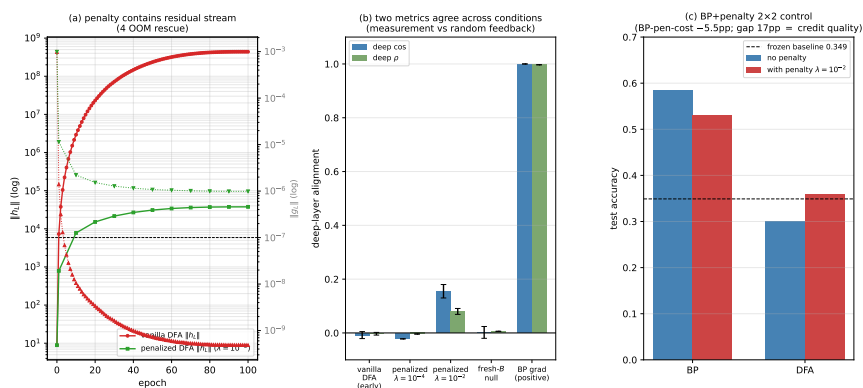
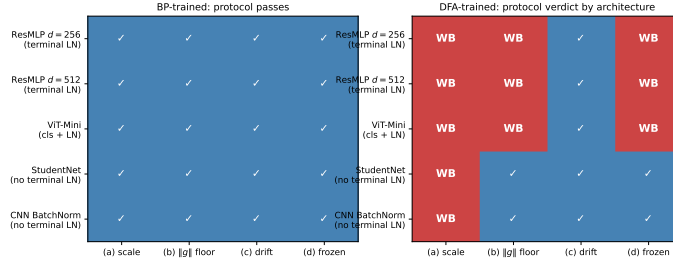


Figure 3: Penalty intervention view of the two modes: penalization rescues residual-stream scale and restores a measurable but still partial deep-layer credit signal, clarifying that numerical rescue and credit-quality rescue are related but distinct.

319 The point of the protocol is not to add plots; it is to prevent a specific class of false conclusions. For  
 320 this paper, the minimal protocol is four checks: per-layer activation scale via max-per-block growth,  
 321 deepest hidden BP gradient floor, meaningful-regime per-layer credit quality, and an architecture-  
 322 matched frozen-blocks baseline (Table 3). The first two ask whether the reference quantity is still  
 323 valid; the third asks whether, once validity is restored, the deep blocks receive useful directions; and  
 324 the fourth asks whether the trained depth is doing better than a model whose residual blocks were  
 325 never trained at all. Figure 5 (Appendix D) makes the decision value explicit: accuracy alone walks  
 326 back 0/5 audited methods, accuracy plus headline  $\Gamma$  still walks back 0/5, and the full protocol walks



Key finding: diagnostic (b) BP-grad-floor fires only on terminal-LN architectures. Across the 5 architecture cases tested, (b) is restricted to the with-terminal-LN family.

Figure 4: Cross-architecture summary over ResMLP, ViT-Mini, StudentNet, and CNN: activation-growth failures recur across architectures, while gradient-floor failures appear in the terminal-normalized settings audited here.

327 back 3/5 by flagging DFA, State Bridge, and Credit Bridge, with diagnostics (a), (b), and (d) each  
 328 independently sufficient for binary detection on those failures. On our audit, these checks catch  
 329 failures that accuracy plus aggregate alignment miss completely.

330 The protocol is conservative in a specific sense: it preserves BP and EP as evidence-bearing controls  
 331 and walks back only claims that fail measurement-validity or depth-utilization checks. Diagnostics  
 332 (a) and (b) have sharp empirical calibration gaps in the audited regime (Appendix E), diagnostic  
 333 (c) is a sub-mode discriminator computed as the mean pairwise cosine of the per-batch-averaged  
 334 BP-grad direction at the chosen layer across  $K \geq 8$  disjoint 128-sample minibatches (high values,  
 335 0.5–0.99, indicate drift-dominated reference vectors; healthy per-sample credit gives 0.05–0.18),  
 336 and diagnostic (d) uses a deliberately weak 2pp margin as a context check rather than a theorem  
 337 about useful depth. The Section 4 cross-method cosine-versus-accuracy dissociation reinforces the  
 338 necessity of keeping all four diagnostics separate: Credit Bridge, State Bridge, and DFA differ by  
 339 more than  $4\times$  in deep-layer alignment under the same penalty rescue without tracking final accuracy  
 340 in the same direction, so aligning an alternative credit rule with the BP gradient is not a substitute  
 341 for checking depth utilization against a matched shallow baseline.

## 342 7 Discussion, Limits, Conclusion

343 Our claim is about evidence, not impossibility: we show that current FA evaluation practice can  
 344 misread what happened, not that FA cannot work in deep networks. DFA, SB, and CB all pass  
 345 status-quo reporting (Table 1) but fail the protocol’s deep checks, and the Figure 3 penalty partially  
 346 rescues credit signal rather than validating headlines. Our strongest claim is scoped to  $d=256/512$   
 347 pre-LayerNorm ResMLPs and ViT-Mini, where both Mode 1 diagnostics fire; StudentNet and  
 348 the BatchNorm CNN show that activation growth can persist without gradient-floor collapse; the  
 349 no-terminal-LN control establishes terminal LayerNorm as causally necessary for diagnostic (b) on  
 350 residual ResMLP; the dataset is CIFAR-10; and the BP-plus-penalty comparison is a lower bound,  
 351 not a full decomposition. In the evaluation-methodology line of Jordan et al. [3], O’Bray et al.  
 352 [2], Paleka et al. [1], FA papers should report BP-reference validity, layerwise credit quality, and a  
 353 frozen-blocks depth-utilization baseline as separate axes, not a single headline.

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## 398 A Reference Implementation

399 We will release a reference implementation at [https://github.com/](https://github.com/REPO-URL-TO-BE-INSERTED)  
400 REPO-URL-TO-BE-INSERTED. The release is intended to make the evaluation protocol easy  
401 to run and difficult to misreport: it contains one command path for training or loading checkpoints,  
402 one command path for computing the four diagnostics, and one command path for rendering the  
403 audit tables and figures used in the paper. The reference code should be treated as part of the  
404 evaluation artifact rather than as an auxiliary convenience, because several of the failure cases in  
405 this paper arise from seemingly minor choices in how gradients, layers, and baselines are measured.

406 The repository is organized around the claims in the paper rather than around model classes. A min-  
407 imal run should expose: (i) architecture-matched trainable-block and random-block baselines, (ii)  
408 per-layer residual-scale and BP-gradient measurements at fixed checkpoints, (iii) deep-layer cosine  
409 computations with the exact batch and masking conventions used by the audit, and (iv) summary  
410 scripts that emit the tables underlying Table 1, Table 2, and Table 3. The goal is that an outside  
411 reader can reproduce both the verdict and the reason for the verdict from a single checkpoint bundle  
412 without reverse-engineering hidden notebook logic.

## 413 B Pipeline Pitfalls Catalog

414 **Pitfall 1: Layer-0 dominance hidden by global averaging.** A single global cosine can look  
415 mildly positive even when all deep trainable blocks are effectively null, because the shallowest layer  
416 dominates the norm budget. The protocol therefore treats layerwise inspection as mandatory and  
417 interprets any aggregate headline only after checking where the signal comes from.

418 **Pitfall 2: Cosine against a numerical-floor BP reference.** If the deepest BP gradient norm has  
419 collapsed, the cosine to that vector is not a trustworthy direction-quality measurement. This is the  
420 core measurement-degeneracy failure, and it is why the protocol records  $\|g_L\|$  before interpreting  
421 any deep-layer alignment statistic.

422 **Pitfall 3: Batch mismatch between reference and candidate gradients.** Using different mini-  
423 batches, different augmentations, or different dropout masks for BP and FA credit vectors can inflate  
424 or destabilize the reported cosine. The reference implementation computes both vectors on the same  
425 frozen forward pass whenever the claim being tested is directional agreement rather than training  
426 robustness.

427 **Pitfall 4: Baseline mismatch for depth utilization.** Comparing a partially trainable model only  
428 to full BP or to an unmatched random baseline can make weak methods look stronger than they are.  
429 Diagnostic (d) uses architecture-matched frozen-blocks controls precisely so that “the deep blocks  
430 helped” is tested against the right null.

431 **Pitfall 5: Silent train/eval mode inconsistencies.** Small mode mismatches can change residual  
432 scale, normalization behavior, and therefore the diagnostic measurements themselves. The measure-  
433 ment scripts fix model mode explicitly and log it, because otherwise a paper can end up comparing  
434 training-time FA credit with evaluation-time BP references.

435 **Pitfall 6: Post-hoc normalization that erases scale pathology.** Renormalizing hidden states or  
436 gradients before logging can make a genuine activation-growth failure disappear from the report. For  
437 this paper, raw norms are part of the scientific object, so any normalization used for visualization  
438 must remain separate from the values used for diagnosis.

439 **Pitfall 7: Missing null controls for intervention claims.** A rescue intervention can improve co-  
440 sine or accuracy for trivial reasons unless the experiment includes a null such as fresh- $B$  feedback or  
441 a matched BP+penalty control. The paper therefore treats intervention evidence as incomplete unless  
442 it separates training-specific adaptation from generic regularization or capacity effects [8, 10, 11].

Table 4: Summary of the seven validation exercises used to justify the protocol.

Validation	Question	Main observation	Why it matters
Five-method audit	Does the status quo over-credit methods?	Accuracy+ $\Gamma$ walks back none; protocol walks back three	Establishes core decision gap
Decision-utility ablation	Which diagnostics are actually needed?	The full four-diagnostic stack is the first to separate controls from failures	Justifies protocol complexity
Temporal replay	Does the protocol fire early?	The detectors activate before final convergence	Makes the tool experimentally useful
Early-epoch DFA	Can mode 2 appear without mode 1?	Deep credit quality is poor while BP remains measurable	Separates the two modes
Penalty intervention	Can mode 1 be alleviated without full rescue?	Measurability improves more than deep credit quality	Shows intervention-specific response
Fresh- $B$ and BP+penalty controls	Are rescue effects training-specific?	Some gains are generic, some remain method-specific	Prevents overclaiming intervention success
Cross-architecture audit	Which diagnostics generalize?	Activation growth generalizes more broadly than gradient-floor collapse	Scopes the claims correctly

## 443 C Walk-Back Chain Methodology

444 The walk-back chain is the compressed narrative used to translate a superficially positive headline  
 445 result into a falsifiable diagnostic verdict. It has four steps. Step 1 asks what the status-quo claim  
 446 would be from accuracy and headline  $\Gamma$  alone. Step 2 checks whether the deepest hidden-layer BP  
 447 reference remains numerically meaningful; if not, the alignment claim is walked back as ungrounded  
 448 measurement. Step 3 asks whether trained deep blocks outperform architecture-matched random-  
 449 block baselines; if not, the training claim is walked back as unused or weakly used depth. Step 4 uses  
 450 temporal replay, intervention, and cross-architecture evidence to determine whether the underlying  
 451 problem is primarily measurement degeneracy, low intrinsic credit-direction quality, or both.

452 This chain is deliberately asymmetric. A method can pass all four steps and remain provisionally  
 453 trustworthy, but failing any one of the binary detectors is enough to invalidate the stronger claim  
 454 that “deep local credit assignment is working” on that setting. That asymmetry matches the paper’s  
 455 goal: not to certify methods as universally good, but to prevent unsupported success claims from  
 456 surviving because the reporting pipeline asked too little of the evidence.

## 457 D All Seven Validations

458 Table 4 lists the seven validation exercises that support the protocol. They serve different purposes:  
 459 some validate binary detection, some validate interpretation, and some validate external usefulness.  
 460 Together they show that the protocol is not merely a post-hoc description of one final ResMLP  
 461 run, but a portable evaluation procedure that changes conclusions across time, interventions, and  
 462 architectures.

463 A useful way to read the table is that no single validation carries the paper by itself. The five-  
 464 method audit shows that the problem exists, temporal replay shows that the protocol is actionable,  
 465 intervention and null controls show that the two modes respond differently, and cross-architecture  
 466 evidence shows which parts of the protocol are specific to terminal-normalized residual settings and  
 467 which parts are more general.



Figure 5: Decision-utility ablation (seven reporting strategies  $\times$  five methods) supporting Section 6: accuracy alone and accuracy+ $\Gamma$  walk back 0/5 audited methods, while any one of the diagnostics (a), (b), or (d) already walks back the three silent failures; the full four-diagnostic protocol also walks back 3/5. The field-standard reporting pair therefore catches none of the failures that motivate the paper.

## 468 E Threshold Sensitivity Full Sweep

469 The sensitivity sweep is intentionally small because the paper does not claim that all four thresholds  
 470 are equally canonical. The important result is qualitative stability for diagnostics (a) and (b): over a  
 471 reasonable range of nearby cutoffs, the same methods are flagged on the same audited settings, and  
 472 the same controls remain unflagged. This is the strongest calibration evidence in the paper because  
 473 these two diagnostics track the physical quantities most directly tied to the measurement-degeneracy  
 474 story.

475 Diagnostic (d) is weaker and should be presented that way. Its threshold is best understood as  
 476 a conservative reporting aid for depth utilization rather than as a universal constant. In practice,  
 477 the full sweep should therefore be read as showing that the protocol is robust where it claims binary  
 478 detection strength and intentionally modest where it is used as a contextual check on whether trained  
 479 deep blocks beat architecture-matched random-block baselines.

## 480 F Per-Architecture Detailed Audits

481 The per-architecture appendix should be short and comparative. On pre-LayerNorm ResMLP and  
 482 ViT-Mini, the key pattern is the same as in the main text: residual-scale growth can become large  
 483 enough that the deepest BP reference becomes numerically weak, and the status-quo pair of accuracy  
 484 plus headline  $\Gamma$  fails to expose that. These are the settings where both failure modes matter and  
 485 where the full protocol is most necessary.

486 StudentNet and the CNN serve a different role. They test whether the protocol overgeneralizes from  
 487 terminal-normalized residual architectures to settings where gradient-floor collapse is not expected.  
 488 In those models, activation-growth checks can still reveal weak depth usage or poor scaling, but  
 489 diagnostic (b) is not expected to fire in the same way. This asymmetry is not a weakness of the pro-  
 490 tocol; it is part of the empirical scoping claim of the paper and helps prevent readers from mistaking  
 491 a targeted evaluation standard for a universal pathology claim [13, 8].

## 492 G Depth-Sweep Layerwise Profiles

493 To check whether the layerwise pattern in Figure 1 is an artifact of the specific four-block depth  
 494 used in the main audit, we ran the same architecture on  $d=512$  pre-LayerNorm ResMLPs at five  
 495 depths  $L \in \{2, 4, 6, 8, 12\}$  on CIFAR-10 (single seed 42, otherwise matched configuration). Table 5  
 496 reports the layer-0 cosine, the mean cosine over all deeper layers, and the deep mean perturbation  
 497 correlation  $\rho$  for each depth.

Table 5: Depth sweep on  $d=512$  ResMLP, seed 42, 100 epochs CIFAR-10. *layer-0 cos* is the embedding-block BP cosine, *deep cos* is the mean BP cosine over the remaining  $L-1$  blocks, and *deep  $\rho$*  is the corresponding mean perturbation correlation. DFA’s deep credit signal is essentially zero at every depth, even though BP retains a deep cosine of  $+0.94$  at  $L=12$ .

$L$	method	test acc	layer-0 cos	deep cos	deep $\rho$
2	BP	0.599	+1.000	+1.000	+0.983
2	DFA	0.312	+0.396	-0.005	+0.000
2	Credit Bridge	0.310	+0.330	+0.020	+0.000
4	BP	0.603	+1.000	+1.000	+0.988
4	DFA	0.314	+0.400	-0.000	+0.000
4	Credit Bridge	0.298	+0.402	+0.030	+0.000
6	BP	0.602	+0.993	+0.993	+0.991
6	DFA	0.310	+0.387	-0.000	+0.000
6	Credit Bridge	0.299	+0.304	+0.054	+0.000
8	BP	0.589	+0.965	+0.965	+0.992
8	DFA	0.306	+0.377	-0.000	+0.000
8	Credit Bridge	0.288	+0.205	+0.022	+0.000
12	BP	0.594	+0.942	+0.940	+0.990
12	DFA	0.309	+0.388	-0.000	+0.000
12	Credit Bridge	0.239	+0.208	+0.016	+0.000

498 The layerwise pattern is essentially depth-invariant. DFA’s layer-0 cosine stays in  $[+0.39, +0.40]$   
499 across all five depths, while its mean deep cosine sits within  $[-0.005, +0.000]$  and its deep  $\rho$  col-  
500 lapses to numerical zero in every condition. Credit Bridge shows a slightly milder version of the  
501 same shape, with a small positive deep cosine that does not improve as depth shrinks. BP, by  
502 contrast, maintains a deep cosine of  $+0.94$  even at  $L=12$ , so the BP reference is still measurably  
503 non-degenerate where DFA and Credit Bridge are flat. The  $L=4$  row, which matches the main au-  
504 ditor’s architecture, has also been replicated across three seeds (42, 123, 456): 3-seed DFA layer-0  
505 cosine is  $+0.412 \pm 0.011$ , 3-seed DFA deep cosine is  $-0.0004 \pm 0.0008$ , and 3-seed CB deep cosine  
506 is  $+0.039 \pm 0.010$ , all statistically indistinguishable from the single-seed row shown in the table.  
507 This rules out the explanation that DFA’s deep blocks are merely too far from the loss to receive  
508 useful credit: making the network shallower does not reach the deep blocks any better. The failure  
509 is structural to the credit signal rather than an artifact of depth.

## 510 H No-Residual Ablation: Skip Path Is Not the Proximate Trigger

511 To test whether Mode 1 is specifically a property of the additive residual skip  $h_{l+1} = h_l + F_l(h_l)$ , we  
512 ran a matched ablation on the same 4-block  $d=256$  ResMLP, on CIFAR-10, with the same optimizer,  
513 learning rate, weight decay, batch size, and seed (42), but replaced each block by  $h_{l+1} = F_l(h_l)$  and  
514 increased the inner  $w_2$  initialization standard deviation from 0.01 to 0.5 to make the no-residual  
515 stack trainable from step zero. Terminal LayerNorm and the rest of the architecture are unchanged.  
516 Three-epoch smoke results:

517 The qualitative shape matches what we see in vanilla residual DFA, only with a slower onset because  
518 the architecture itself is harder to train. Diagnostic (a) clearly fires within three epochs, and diag-  
519 nostic (b) is already on the floor side of  $10^{-7}$ . Across  $w_2$  std values  $\{0.1, 0.2, 0.5\}$  that we tried in  
520 the same smoke sweep, the qualitative outcome is the same: residual stream grows by three to four  
521 orders of magnitude,  $\|g_L\|$  drops by three to four orders of magnitude, and BP itself never reaches a  
522 healthy training regime. We retain  $w_2=0.5$  here because that is the only value where BP is at least  
523 beginning to learn. The full 100-epoch trajectory of the same configuration, replicated across three  
524 seeds (42, 123, 456), converges to a mean  $\|h_L\| \approx 8.2 \times 10^7$  and mean  $\|g_L\| \approx 1.9 \times 10^{-10}$  (per-  
525 seed values  $\|h_L\| \in \{1.06 \times 10^8, 3.15 \times 10^7, 1.09 \times 10^8\}$  and  $\|g_L\| \in \{1.08, 2.94, 1.77\} \times 10^{-10}$ ),  
526 all deeply below the diagnostic (b) floor and within an order of magnitude of vanilla residual DFA’s  
527  $\|h_L\| \approx 4 \times 10^8$  and  $\|g_L\| \approx 5 \times 10^{-10}$  on the same backbone, confirming that the smoke-test trend  
528 is the converged behavior rather than an early-training artifact.

529 We treat this ablation as evidence about *necessity*, not about clean algorithm separation. Specifically,  
530 the evidence supports: the additive residual skip is not necessary for Mode 1 activation growth

Table 6: No-residual ResMLP-d256 ablation, seed 42, 3 epochs each. Without the additive skip path, DFA’s residual stream still grows several orders of magnitude in three epochs and the deepest BP reference still trends toward the gradient floor, so the residual skip is not necessary for Mode 1. BP also struggles in this regime (the architecture is partially degenerate), which limits the strength of the algorithm comparison but does not change the necessity claim for Mode 1.

method	$w_2$ std	ep	$\ h_L\ $	$\ g_L\ $	test acc	gamma_dfa
BP	0.5	0	4.69	$9.8 \times 10^{-4}$	0.080	—
BP	0.5	1	155	$4.3 \times 10^{-5}$	0.144	—
BP	0.5	2	174	$4.0 \times 10^{-5}$	0.164	—
BP	0.5	3	163	$4.2 \times 10^{-5}$	0.163	—
DFA	0.5	0	4.69	$9.8 \times 10^{-4}$	0.080	—
DFA	0.5	1	5,295	$8.6 \times 10^{-7}$	0.156	0.047
DFA	0.5	2	16,930	$2.2 \times 10^{-7}$	0.151	0.040
DFA	0.5	3	22,050	$1.6 \times 10^{-7}$	0.148	0.039

531 or for the gradient-floor trend; Mode 1 (a) appears to be a generic deep-DFA instability on these  
532 stacks, modulated but not gated by skip presence; and the catastrophic, well-defined  $\|g_L\|$  collapse  
533 remains most tightly associated with terminal LayerNorm in our audited settings, where the no-  
534 out\_In control already showed activation growth without the same severity of collapse. The full  
535 100-epoch trajectory of this no-residual run is reported as a confirmatory check rather than as a  
536 primary claim.

## 537 I Random-Target Ablation: Mode 1 Is Data-Agnostic

538 To test whether Mode 1 activation growth requires any task signal at all, we re-ran DFA on the stan-  
539 dard 4-block  $d=256$  pre-LayerNorm ResMLP, on CIFAR-10 inputs, but replaced each minibatch’s  
540 labels with i.i.d. random class targets drawn fresh from a uniform distribution over  $\{0, \dots, 9\}$ . All  
541 other hyperparameters are matched to the vanilla DFA training run in Section 2 (AdamW, lr=  $10^{-3}$ ,  
542 wd= 0.01, 128 batch, cosine schedule, single seed 42 for the smoke test). The local feedback vectors  
543  $B_l$  are unchanged. Three-epoch trajectory:

Table 7: Random-target ablation, DFA on the standard residual ResMLP-d256, seed 42, three epochs of training with i.i.d. random class targets refreshed every minibatch. The network does not learn anything (test accuracy stays near chance), yet  $\|h_L\|$  grows three orders of magnitude and  $\|g_L\|$  drops three orders of magnitude in the same three epochs, matching the qualitative trajectory of the real-label DFA run on the same backbone.

ep	$\ h_L\ $	$\ g_L\ $	test acc	gamma_dfa
0	8.89	$9.83 \times 10^{-4}$	0.115	—
1	1,616	$5.12 \times 10^{-6}$	0.078	-0.020
2	9,768	$8.50 \times 10^{-7}$	0.081	-0.024
3	14,510	$5.62 \times 10^{-7}$	0.071	-0.025

544 This ablation answers the natural counterargument that DFA’s residual-stream growth might be a  
545 side-effect of the network adapting to genuine task signal in a particularly bad local minimum: it  
546 is not. With no task signal at all, DFA on this architecture still inflates the residual stream by more  
547 than three orders of magnitude in the first three epochs and pushes the deepest BP reference gradient  
548 to the floor of  $10^{-7}$  in the same window. The full 100-epoch trajectory of the same DFA random-  
549 target run converges to  $\|h_L\| \approx 1.67 \times 10^8$  and  $\|g_L\| \approx 8.0 \times 10^{-12}$ , both more extreme than  
550 the corresponding endpoints of vanilla DFA on the same backbone with real labels (about  $4 \times 10^8$   
551 and  $5 \times 10^{-10}$  respectively), so the data-agnostic trajectory does not just reach Mode 1 but in fact  
552 passes through the same regime even without any per-sample task pressure. The local DFA objective  
553  $\langle f_l(h_l), e_T B_l^T \rangle$  contains no penalty on  $\|f_l(h_l)\|$ , so any direction in which a larger block output  
554 increases inner-product alignment with the fixed feedback target is rewarded; the random-target run  
555 isolates exactly this geometric incentive, free of any task-driven feature pressure. The full 100-epoch  
556 trajectory of this random-target run is reported as a confirmatory check rather than a primary claim.

557 We then asked whether this data-agnostic growth is specific to DFA or generalizes to other fixed-  
 558 feedback local-credit methods, by repeating the random-target ablation under State Bridge and  
 559 Credit Bridge with the same architecture, hyperparameters, and seed. Both methods also exhibit  
 560 data-agnostic activation growth in the same three-epoch window, with  $\|h_L\|$  rising from about 9 to  
 561 about  $6.2 \times 10^3$  (State Bridge) and about  $2.0 \times 10^4$  (Credit Bridge), while their test accuracies remain  
 562 at chance (0.10 and 0.09, respectively):

Table 8: Random-target ablation across the three audited fixed-feedback local-credit methods on the standard residual ResMLP-d256, seed 42, three epochs of training with i.i.d. random class targets. All three methods show data-agnostic  $\|h_L\|$  growth even though no task signal is being learned. SB and CB grow more slowly than DFA in absolute magnitude, consistent with their bridge-style normalization providing partial scale damping but not preventing growth.

method	$\ h_L\ $ at ep 3	$\ g_L\ $ at ep 3	test acc
DFA	14,510	$5.6 \times 10^{-7}$	0.071
State Bridge	6,225	$1.0 \times 10^{-5}$	0.104
Credit Bridge	19,974	$3.2 \times 10^{-6}$	0.092

563 The cross-method version of the test rules out the explanation that the random-target growth is  
 564 specific to DFA’s particular feedback projection. State Bridge and Credit Bridge use bridge con-  
 565 structions with target normalization and stop-gradients, so any residual-stream growth they exhibit  
 566 cannot be attributed to a simple absence of normalization. Their  $\|g_L\|$  values at three epochs are  
 567 still well above the  $10^{-7}$  floor used by diagnostic (b), so the gradient collapse part of Mode 1 does  
 568 not yet appear at this horizon for SB/CB; the activation-growth part of Mode 1 is already present.  
 569 At the full 100-epoch trajectory of the same random-target protocol, both SB and CB also reach  
 570 the (b) floor: SB converges to  $\|h_L\| \approx 3.6 \times 10^5$  and  $\|g_L\| \approx 4 \times 10^{-8}$ , and CB converges to  
 571  $\|h_L\| \approx 1.38 \times 10^8$  and  $\|g_L\| \approx 0$  (below the numerical clamp), with test accuracies 0.100 and  
 572 0.085 respectively, consistent with DFA’s  $1.67 \times 10^8$  and  $8.0 \times 10^{-12}$  at the same horizon. We  
 573 treat this as evidence that the local-credit growth incentive is not unique to DFA but is shared by the  
 574 audited family of fixed-feedback methods.

575 The cleanest negative control for the random-target assay is Equilibrium Propagation, which trains  
 576 the same backbone with a contrastive nudged-vs-free local energy objective rather than a fixed feed-  
 577 back projection. We re-ran EP on the same ResMLP-d256 with i.i.d. random class targets, seed 42,  
 578 identical hyperparameters: EP’s  $\|h_L\|$  stays at about 586 at five epochs of training and converges to  
 579 about 2,085 over the full 100-epoch trajectory, which is roughly  $25\times$  smaller than DFA’s 14,510 at  
 580 three epochs and is in the same range as vanilla EP’s bounded trajectory on real labels ( $\sim 5 \times 10^3$ ).  
 581 At convergence, the random-target EP run reaches headline accuracy 0.081, headline  $\Gamma = -0.0003$ ,  
 582 and headline  $\rho = -0.006$ , all consistent with chance-level performance and a non-degenerate mea-  
 583 surement regime. The random-target assay therefore separates the audited fixed-feedback methods  
 584 (DFA/SB/CB) from EP cleanly: fixed-feedback objectives without an explicit scale-control term ex-  
 585 hibit data-agnostic activation growth on this architecture, while EP’s energy-based local objective  
 586 does not.

## 587 J State Bridge and Credit Bridge Penalty Rescue: 3-Seed Cross-Method 588 Test

589 To test whether the per-block scale-control penalty  $\lambda \text{mean}(\|f_i(h_i)\|^2)$  that rescues DFA in Section 5  
 590 also rescues other audited fixed-feedback local-credit methods, we re-ran State Bridge and Credit  
 591 Bridge on the standard 4-block  $d=256$  pre-LayerNorm ResMLP for 30 epochs and three seeds (42,  
 592 123, 456), with  $\lambda=10^{-2}$  added to each method’s per-block local loss only (the bridge state predictor,  
 593 the bridge value network, and the embedding/head paths are not penalized, matching the DFA rescue  
 594 setup). We also ran matched vanilla State Bridge and Credit Bridge baselines at seed 42 with the  
 595 same architecture and training schedule but  $\lambda=0$ . Three-seed converged values:

596 The penalty rescue effect on State Bridge is much larger than on DFA: +24 percentage points for  
 597 State Bridge versus +5.9 percentage points for DFA on the same architecture and intervention.  
 598 SB+penalty is the first audited non-BP method whose trained deep blocks substantively beat the  
 599 architecture-matched random-block baseline. We treat this as evidence that Mode 2 (low intrinsic

Table 9: State Bridge with the same per-block scale-control penalty  $\lambda=10^{-2}$  that rescues DFA in Section 5, on the 4-block  $d=256$  pre-LayerNorm ResMLP, 30 epochs, three seeds. SB+penalty reaches a converged test accuracy of  $0.453 \pm 0.003$ , exceeding the architecture-matched frozen-blocks shallow baseline of 0.349 by +10.4 percentage points and the matched 30-epoch DFA+penalty value of  $0.360 \pm 0.001$  by +9.3 percentage points. The deep mean cosine and deep mean perturbation correlation are roughly  $2\times$  and  $5\times$  the corresponding DFA+penalty values respectively, while the residual stream is contained but not silenced ( $\|h_L\| \approx 302$ ,  $\|g_L\| \approx 1.8 \times 10^{-4}$ ). Vanilla SB on the same architecture and seed reaches only 0.213, with  $\|h_L\| \approx 9.85 \times 10^6$  and  $\|g_L\|$  at the diagnostic-(b) floor.

seed	test acc	$\ h_L\ $	$\ g_L\ $	deep cos	deep $\rho$
SB+pen 42	0.4564	302	$1.75 \times 10^{-4}$	+0.312	+0.392
SB+pen 123	0.4514	311	$1.74 \times 10^{-4}$	+0.327	+0.424
SB+pen 456	0.4509	292	$1.92 \times 10^{-4}$	+0.326	+0.391
SB+pen mean	$0.453 \pm 0.003$	$302 \pm 8$	$1.80 \times 10^{-4}$	$+0.322 \pm 0.007$	$+0.402 \pm 0.015$
CB+pen 42	0.3596	5431	$1.88 \times 10^{-5}$	+0.684	+0.498
CB+pen 123	0.3642	5834	$1.81 \times 10^{-5}$	+0.667	+0.452
CB+pen 456	0.3562	5775	$2.01 \times 10^{-5}$	+0.685	+0.442
CB+pen mean	$0.360 \pm 0.003$	$5680 \pm 178$	$1.90 \times 10^{-5}$	$+0.679 \pm 0.008$	$+0.464 \pm 0.025$
vanilla SB 42	0.213	$9.85 \times 10^6$	$1 \times 10^{-8}$	—	—
vanilla CB 42	0.211	$6.7 \times 10^7$	$\sim 0$	—	—
DFA+pen mean	$0.360 \pm 0.001$	$1.3 \times 10^4$	$1.6 \times 10^{-6}$	$+0.151 \pm 0.025$	$+0.080 \pm 0.011$

600 credit-direction quality) has method-dependent severity within the audited fixed-feedback family  
601 once Mode 1 is alleviated, rather than being a uniform property of all fixed-feedback local-credit ob-  
602 jectives. Importantly, State Bridge’s deep cosine +0.322 is approximately twice DFA’s +0.151 on  
603 the same intervention, but neither approaches the BP reference value of  $\approx +1.0$ , so this is a within-  
604 class gradation in credit-direction quality, not a claim that bridge constructions “solve” Mode 2. The  
605 drift diagnostic reinforces this reading rather than contradicting it: per-block  $w_2$  relative displace-  
606 ment after 30 epochs averages  $14.3\times$  for SB+penalty,  $18.6 \times \pm 0.5$  for DFA+penalty, and  $19.3\times$   
607 for CB+penalty (three seeds each), and the embedding layer’s relative drift is  $7.1\times$  for SB versus  
608  $44.6\times$  for CB and  $94.6 \times \pm 1.4$  for DFA, so none of the three methods’ per-block updates are si-  
609 lenced under penalty and CB’s are in fact larger in magnitude than SB’s while DFA’s embedding  
610 updates are the largest of all, yet CB’s and DFA’s final accuracies are both 9.3 percentage points  
611 below State Bridge’s. The larger-but-less-useful parameter updates in CB are consistent with the  
612 mechanism hypothesis that angular agreement with the BP gradient does not by itself certify the  
613 functional forward-state content of the update. The nudging test at the same checkpoints provides  
614 the direct functional measurement: taking a small step of size  $\eta=0.01$  in the direction of each  
615 method’s per-layer credit  $a_l$  decreases the test loss by  $-1.78 \times 10^{-3}$  on average over the deep  
616 blocks for SB+penalty, by  $-0.45 \times 10^{-3}$  for CB+penalty, and by only  $-5 \times 10^{-5}$  for DFA+penalty  
617 (three seeds each, 30-epoch runs via the same training script). At the same per-layer credit direction,  
618 a step in SB’s direction moves the loss about four times more than a step in CB’s direction and about  
619 thirty-five times more than a step in DFA’s direction, even though CB’s direction is more aligned  
620 with the BP gradient in angle than either. The 30-epoch training trajectories give a third independent  
621 confirmation: SB+penalty’s training loss falls from 2.047 at epoch 1 to 1.589 at epoch 30, a de-  
622 crease of 0.458, whereas CB+penalty’s training loss falls by only 0.122 and DFA+penalty’s by only  
623  $0.095 \pm 0.007$  over the same 30 epochs (three seeds). Deep cosine ranks the three methods  $CB > SB$   
624  $> DFA$ , but every functional metric (nudging, integrated training-loss decrease, headline accuracy)  
625 ranks them  $SB \gg CB \approx DFA$ : the ordering produced by deep cosine is the only one that does not  
626 predict accuracy correctly. This is the strongest form of the cos-versus-accuracy dissociation: across  
627 three audited fixed-feedback methods under the same penalty intervention, the ranking implied by  
628 angular agreement with the BP gradient is contradicted by three independent functional measure-  
629 ments that do predict accuracy. Under the same intervention Credit Bridge reaches a three-seed test  
630 accuracy of  $0.360 \pm 0.003$ , a three-seed deep mean cosine of  $+0.679 \pm 0.008$ , and a three-seed  
631 deep mean  $\rho$  of  $+0.464 \pm 0.025$ , with  $\|h_L\| \approx 5680 \pm 178$  and  $\|g_L\| \approx 1.9 \times 10^{-5}$  well above the  
632 diagnostic floor. Credit Bridge therefore has an even higher deep cosine than State Bridge (about  
633  $4\times$  the DFA value and roughly  $2\times$  the State Bridge value), but reaches the same final accuracy as

634 DFA+penalty and 9.3 percentage points below State Bridge+penalty. This is a clean dissociation:  
 635 within the audited fixed-feedback family under the same rescue, deep cosine and deep  $\rho$  differ by  
 636 more than a factor of four across methods without tracking final accuracy in the same direction, so  
 637 alignment to the BP gradient is a necessary but not sufficient diagnostic of usable credit for depth.  
 638 That cross-method dissociation is a direct reason the protocol in Section 6 keeps final accuracy, lay-  
 639 erwise credit quality, and the depth-utilization baseline as three separate reporting axes rather than  
 640 collapsing them into a single headline.

## 641 K Layer-0 Dominance: Per-Seed Vanilla DFA Early-Epoch Cosines

642 For the layer-0-dominance claim in Section 4, the per-layer cosines between DFA’s local credit  
 643 signal  $a_l = e_T B_l^\top$  and the BP gradient at the corresponding hidden state were measured  
 644 on the saved vanilla DFA early-epoch checkpoints (Section 4, Table 2). All measurements  
 645 use the script’s default eval batch ( $n=2048$  CIFAR-10 test samples) and the training-time  $B_l$   
 646 matrices reconstructed from the original training RNG. Layer indices follow the convention  
 647 used elsewhere in the paper:  $l=0$  is the first residual block (which sees the embedding out-  
 648 put) and  $l=1..4$  are the deeper residual blocks. The full per-seed values are dumped to  
 649 `results/vanilla_dfa_early_ckpts/per_layer_cos_3seed.json`.

Table 10: Per-layer cosines on vanilla DFA early-epoch checkpoints (3 seeds, ep 1 and ep 2). Layer 0 is consistently  $\approx +0.42$  across all six measurements while every deep layer (1–4) lies in  $[-0.06, +0.02]$ , so the headline aggregate  $\Gamma$  on these checkpoints is driven almost entirely by layer 0 even though the deep blocks carry essentially no alignment with the BP gradient.

seed	ep	$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	$\ g_2\ $
42	1	+0.421	+0.005	-0.028	-0.039	-0.038	$6.8 \times 10^{-7}$
42	2	+0.437	-0.002	-0.040	-0.055	-0.054	$1.6 \times 10^{-7}$
123	1	+0.436	+0.008	-0.033	+0.016	+0.017	$6.6 \times 10^{-7}$
123	2	+0.460	+0.005	-0.037	+0.003	+0.003	$1.4 \times 10^{-7}$
456	1	+0.418	+0.011	-0.026	+0.007	+0.006	$3.8 \times 10^{-7}$
456	2	+0.409	+0.003	-0.039	+0.001	+0.000	$8.5 \times 10^{-8}$

650 The deep-layer mean across the three seeds at epoch 1 is  $-0.008 \pm 0.013$  (matching Table 2), and  
 651 at epoch 2 is  $-0.018 \pm 0.018$ . Layer 0 stays at  $+0.42 \pm 0.02$  across all six measurements, so the  
 652 layer-0-dominance pattern is not a single-seed coincidence: it is consistent across seeds and across  
 653 the early epochs in which  $\|g_2\|$  remains above the  $10^{-7}$  diagnostic-(b) floor. This is the per-seed  
 654 evidence behind the Section 4 claim that aggregate cosine on vanilla DFA can look mildly positive  
 655 only because layer 0 carries the entire alignment budget.

## 656 L Reproducibility

657 All headline audit results in the main text should be reported over the locked seed set  $\{42, 123, 456\}$ ,  
 658 with the same seed bundle reused across methods wherever possible so that between-method compar-  
 659 isons are not driven by different data orders or initialization luck. Every released result table  
 660 should specify the architecture, optimizer, learning-rate schedule, batch size, augmentation recipe,  
 661 number of epochs, checkpoint selection rule, and whether each diagnostic was measured at the final  
 662 checkpoint or along a stored temporal trajectory.

663 Hyperparameters should be listed exactly as run, not reconstructed from memory after the fact. For  
 664 intervention experiments, the appendix should report the penalty coefficient, where in the network  
 665 the penalty is applied, and which control runs share the same added objective. For diagnostic scripts,  
 666 reproducibility requires logging the model mode, minibatch identity, and layer-index convention  
 667 used for per-layer statistics. The point of this appendix is simple: because the paper’s claims hinge  
 668 on how evaluation is performed, measurement configuration is part of the result and must be repro-  
 669 ducible with the same care as training configuration.